Lecture 10	**Inner product**: (definition and examples of inner product space)

Lecture 10

INNER PRODUCT SPACES

Topic Objectives: Basic objective of this topic is to make students understand

- the meaning of an inner product space
- prove some properties of inner product spaces

1.1 Introduction:

The inner product generalizes the notion of dot product of vectors in R^n .

Let V be a real vector space. Suppose to each pair of vectors u and v in V, there is assigned a

real number, denoted by $<u , v>$. This function is called a (real) inner product on V if it

satisfies the following axioms:

1. (Linear Property): $<au1+bu2, v> = a<u1, v> +b <u2, v>$.
2. (Symmetric Property): $<u, v> = <v, u>$
3. (Positive Definite Property): $<u , u> \geq 0$; $<u, u> = 0$ iff $u = 0$.

The Vector space V with an inner product is called a (real) inner product space.

Example 1: Let V be a real inner product space, then by Linearity,

Example 2: Consider the vector space R^n. The dot product or scalar product in R^n is defined by

For

This function defines an inner product on R^n.

The vector space R^n with the above inner product and norm is called Euclidean n-space. Although there are many ways to define an inner product on R^n, we shall assume this inner product unless otherwise stated or implied. It is called the usual (or standard) inner product on R^n.

1.2 Cauchy–Schwarz Inequality

Example:

1.3 Properties of norm:

1.4 Examples:

Verify that the following defines an inner product in R^2

Because A is real and symmetric, we need only show that A is positive definite. The diagonal elements 1 and 3 are positive, and the determinant ||A|| = 2 is positive. Thus, A is positive definite. Consequently, < u , v> is an inner product.

Summary

An inner product is a function on a vector space which satisfies the four axioms. The norm or length of a vector denoted by ||u||, is defined as ||u|| =

Homework

1. For an inner product space, <u, v> is equal to <v, u>
   (a) True (correct answer)
   (b) False

2. Which of the following is not an axiom for an inner product space?
   (a) Associativity (correct answer)
   (b) Linearity
   (c) Symmetricity

3. Norm of a vector is ……………
   (a) Always positive.
   (b) Either positive or zero. (correct answer)
   (c) Any real number.
   (d) Always zero.

Applications in Field of Engineering

Some of the main applications of Inner products are

- Study of vectors in the Euclidean space.
- The Frobenius inner product for matrices.
- Applications in Fourier analysis to define Fourier coefficients for the series.
- Applications in boundary value problems (mainly heat and wave equations)

Frequently Asked Questions

Question 1: What is a normed vector space?

Answer: A normed vector space or normed space is a vector space over the real or complex numbers, on which a norm is defined.

Question 2: How is inner product different from dot product?
Answer: An inner product is a generalization of the dot product. In a vector space, it is a way to multiply vectors together, with the result of this multiplication being a scalar.

References

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